

# Calculus 1

Final Exam

October 28, 2025 (18:15 – 20:15)



university of  
 groningen

**Please read the instructions!**

1) Apply L'Hospital's Rule to evaluate the limit  $\lim_{x \rightarrow a} \frac{a^x - x^a}{x - a}$ , where  $a > 0$ . Indicate the results (e.g. limit laws, continuity, differentiation rules) used in each step.

2) Use Taylor Series to find the limit  $\lim_{x \rightarrow \infty} (\sqrt[6]{x^6 + x^5} - \sqrt[6]{x^6 - x^5})$ .

3) Calculate the arc length of the curve  $y = \ln(\sin x)$ ,  $\frac{\pi}{6} \leq x \leq \frac{\pi}{2}$ .

4) Evaluate the integral  $\int_0^\infty e^{-Ax} \cos x \, dx$  in terms of the constant  $A > 0$ .

5) Solve the initial value problem  $y'(x) + x y(x) = x$ ,  $y(0) = 2$ .

6) Solve the following initial value problem

$$y''(x) + 6y'(x) + 9y(x) = 0, \quad y(0) = 1, \quad y'(0) = 1.$$

## Instructions

- **write your name and student number on the top of each sheet of writing paper!**
- use the writing (lined) and scratch (blank) paper provided, raise your hand if you need more paper
- start each question on a new page
- use a pen with black or blue ink
- do not use any kind of correcting fluid or tape
- any rough work should be crossed through neatly so it can be seen
- this is a closed-book exam (books & notes are not permitted)
- you may use your own handwritten formula sheet (a 2-sided A4 paper) – the invigilators will check these
- you are also allowed to use a simple pocket calculator
- programmable/graphing calculators are not allowed
- your work should be clearly and logically structured
- **explain your reasoning using words**
- show all your calculations, an answer without any computation will not be rewarded
- each problem is worth 15 points
- upon completion<sup>1</sup> submit your worksheets at the front desk

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<sup>1</sup>At the end of the exam or after you finished whichever is sooner.